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Vibration of Plate with Thermal Effect and Circular Thickness variation without any Load

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ABSTRACT: An analytical approach for free mechanical vibration analysis of four edges simply supported rectangular Kirchhoff plates is presented. The classical (thin) plate theory is used to analyze vibration of plates in the present study. In present paper a simple model is presented to study the effect of temperature with circular thickness variation on a visco-elastic plate. A projected but suitable frequency equation is resulting by Rayleigh-Ritz method with two term deflection function. The frequencies corresponding to the first two modes of vibration has been calculated for a simple supported visco-elastic plate for various values of taper constant and thermal gradient.

Keywords: Vibration; plate; thickness; taper constant; thermal; clamped

INTRODUCTION

In the engineering, all machines and engineering structures experiences vibrations so we cannot move without considering the effect of vibration. With the advancement of technology, the necessity to know the effect of temperature on visco-elastic plates of variable thickness has become crucial. Tapered Plates with uniform and non-uniform thickness and temperature are widely used in marine structure, aeronautical field, power plants, automobile sector etc. Various researchers studied the vibration behavior of homogeneous or non-homogeneous plates with variable thickness, with or without consideration of temperature effects. An extensive review on linear vibration of plates has been given by Leissa [1] in his monograph and a series of review articles [2]. Tomar and Gupta [3] studied the effect of taper constants in two directions on elastic plates, but not on visco-elastic plates. Bhatnagar and Gupta [4] studied the effect of thermal gradient on vibration of a visco-elastic circular plate of variable thickness. Gupta and Khanna [5] studied the effect of linearly varying thickness in both directions on vibration of a visco-elastic rectangular plate. Gupta and Khanna [6] studied the Vibration of clamped visco-elastic rectangular plate with parabolic thickness variations. Gupta and Khanna [7] analyzed free vibration of clamped rectangular plate with bi-direction exponentially thickness variations. Khanna and Sharma [8] have been studied on Vibration Analysis of Visco-Elastic Square. Plate of Variable Thickness with Thermal Gradient. Khanna & Sharma [9] studied natural vibration of visco-elastic plate of varying thickness with thermal effect. Khanna & Sharma [10] studied Analysis of free vibrations of visco-elastic square plate of variable thickness with temperature effect. Khanna & Sharma [11] analyzed a computational prediction on vibration of square plate by varying thickness with bi-dimensional thermal effect. Sharma, Raghav & Sharma [12] presented the study of A Modeling on frequency of Rectangular Plate. Sharma, Raghav & Sharma [13] represented the Vibrational study of Square Plate with Thermal Effect and Circular Variation in density. In present paper, the authors have studied the thermal effect on the circular vibration of visco-elastic square plate whose thickness and thermal effect vary linearly in x direction. Also, it is supposed that the plate is clamped on all the four edges. Due to temperature deviation, we suppose that non homogeneity occurs in modulus of elasticity. Frequency for the first two modes of vibration is obtained for various numerical values of thermal gradient, tapering constant and non-homogenous constant. Results are presented in graphical and tabular form.

ANALYSIS OF THE MODEL AND SOLUTION

The plate's circular thickness for the present study is to be assumed linearly in x –axis which is represented by

$$j = j_0\left[1+\beta\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right] \qquad (1)$$

where β is known as tapering constant and thickness becomes constant at $\beta = 0$.

One dimensional variation in temperature along x-axis is given by

$$\tau = \tau_0\left(1-\frac{x}{a}\right) \qquad (2)$$

where 'a' represents length of the plate and τ_0 is temperature at origin of the plate.

The temperature dependent modulus of elasticity is taken as

$$E(\tau) = E_0\,(1-\gamma\,\tau) \qquad (3)$$

where E_0 is the Young's modulus and γ is taken as slope variation.
From equation (2) and (3) , we have

$E = E_0 [1 - \gamma\, \tau_0 (1 - \frac{x}{a})]$

$E = E_0 [1 - \alpha (1 - \frac{x}{a})]$ (4)

Where $\alpha = \gamma\, \tau_0$, ($0 \leq \alpha < 1$) is thermal gradient.
For non-homogeneous material , circular variation taken in Poisson ratio is

$h = h_0[1 - c_1\sqrt{1 - \frac{x^2}{a^2}}\,]$ (5)

where c_1 ($0 \leq c_1 < 1$) is non-homogeneity constant.
Four sided clamped edges boundary conditions for the square geometry of plate are taken as

$$\left.\begin{aligned} W = W_x = 0 \text{ at } x = 0, a \\ W = W_y = 0 \text{ at } y = 0, a \end{aligned}\right\} \quad (6)$$

Deflection function, satisfying the boundary conditions, can be taken as

$$W = [\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{a}\right)]^2\ [A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{a}\right)] \quad (7)$$

where A_1 , A_2 are constants to satisfy boundary conditions.

Now, unitless variables having no dimension are using for our convince as

$$X = \frac{x}{a}, Y = \frac{y}{a}, \overline{W} = \frac{W}{a}, \overline{l} = \frac{l}{a} \quad (8)$$

Solution by Rayleigh-Ritz Method: Rayleigh – Ritz method is used to find an appropriate vibrational frequency. This method works on the phenomena that maximum strain energy (P_E) must equal to maximum kinetic energy (K_E). An equation in the following form is obtained as

$$\delta(P_E - K_E) = 0 \quad (9)$$

The expression for Strain Energy and Kinetic are

$$P_E = \frac{1}{2}\int_0^a\int_0^a D\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2h\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 2(1-h)\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right] dx\,dy \quad (10)$$

$$K_E = \frac{1}{2}\ p^2\rho\int_0^a\int_0^a j\, W^2 dx\,dy \quad (11)$$

where $D = \frac{Ej^3}{12(1-h^2)}$ is plate's flexural rigidity and ρ is density of the plate.
Now using the values of j , E and h from equations (1) , (4) and (5) in 'D' , We get

$$D = \frac{E_0 j_0{}^3\ [1 - \alpha\,(1 - \frac{x}{a})](1+\beta\left(1-\sqrt{1-\frac{x^2}{a^2}}\right))^3}{12[1-h_0{}^2\{\ 1 - c_1\ \sqrt{1-\frac{x^2}{a^2}}\,\}^2]} \quad (12)$$

Substitute the values from equations (1), (5), (10), (11) and (12) in equation (9), we get

$$\delta(P_E^* - \lambda^2 K_E^*) = 0 \quad (13)$$

Where,

$$P_E^* = \int_0^a\int_0^a \left\{\frac{E_0\, j_0{}^3\left[1 - \alpha\left(1-\frac{x}{a}\right)\right]\left(1+\beta\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right)^3}{24\left[1-h_0{}^2\{\ 1 - c_1\ \sqrt{1-\frac{x^2}{a^2}}\,\}^2\right]}\right\} \begin{bmatrix} \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + \\ 2h_0\{\ 1 - c_1\ \sqrt{1-\frac{x^2}{a^2}}\,\}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} \\ +2\begin{pmatrix} 1 - h_0\{\ 1\ - \\ c_1\ \sqrt{1-\frac{x^2}{a^2}}\,\} \end{pmatrix}\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2 \end{bmatrix} dx\,dy \quad (14)$$

$$K_E^* = \frac{1}{2}\ p^2\rho\, j_0\int_0^a\int_0^a\ (1 + \beta\left(1 - \sqrt{1-\frac{x^2}{a^2}}\right))\ \ W^2 dx\,dy \quad (15)$$

and frequency $\lambda^2 = \frac{12p^2\rho_0\, a^4}{E_0\, j_0^2}$

Now, the value of A_1 & A_2 is to be determined from (13)

$$\frac{\partial(P_E^* - \lambda^2 K_E^*)}{\partial A_n} = 0 \quad , \qquad \text{for n= 1, 2} \qquad (16)$$

On simplifying (16), we get

$$mn_1A_1 + mn_2A_2 = 0, \quad \text{for n= 1, 2} \qquad (17)$$

Where mn_1, mn_2 (n = 1,2) comprises parametric constant and the frequency parameter.
For non-trivial solution, the determinant of the coefficient of equation (17) must be zero. So, we get the frequency equation as

$$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = 0 \qquad (18)$$

With the help of equation (18), we get a quadratic equation in λ^2 from which the two values of λ^2can be found. These two values represent the first two modes of vibration of frequency i.e. λ_1 (first mode) and λ_2 (second mode) for different values of taper constant and thermal gradient for a clamped plate.

RESULTS AND DISCUSSION

The frequency (λ) for 1st and 2nd mode of vibration of a clamped square plate has been calculated for different values of thermal gradient (α) , taper constant (β) and non-homogeneity constant (c_1). All the results are calculated with the help of MATLAB / MAPPLE software. Following parameters are used for these calculations:

$E_0 = 7.08*10^{10}$ N/ M^2, $h_0 = 0.345$, $\rho_0 = 2.80*10^3$ Kg / M^3, $j_0 = 0.01$m

The results are shown in Tables 1-4 and Figures 1-4.
It is evident from Table 1that as value of thermal gradient (α) increases from 0 to 1 corresponding value of frequency (λ) decreases for both modes of vibration. Further, it is evident from Table 2 that as value of thermal gradient (α) increases from 0 to 1 corresponding value of frequency (λ) decreases for both modes of vibration. Also Table 3 reveals that as value of taper constant (β) increases from 0 to 1 corresponding value of frequency (λ) also increases for both modes of vibration. Table 4 reveals that as value of taper constant (β) increases from 0 to 1 corresponding value of frequency (λ) also increases for both modes of vibration.

Table 1: Thermal Gradient (α) versus frequency (λ) with fixed value of Poisson ratio (h = 0.345) and three different values of taper constant (β = 0, 0.5, 1)

α	$C_1 = \beta = 0$		$C_1 = \beta = 0.5$		$C_1 = \beta = 1$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	38.33	149.98	52.66	205.29	45.90	179.29
0.2	36.36	142.28	50.13	195.47	44.14	172.70
0.4	34.28	134.15	47.46	185.12	42.30	165.87
0.6	32.07	125.48	44.64	174.17	40.37	158.74
0.8	29.69	116.17	41.63	162.48	38.31	151.28
1.0	27.09	106.05	38.37	149.88	36.13	143.44

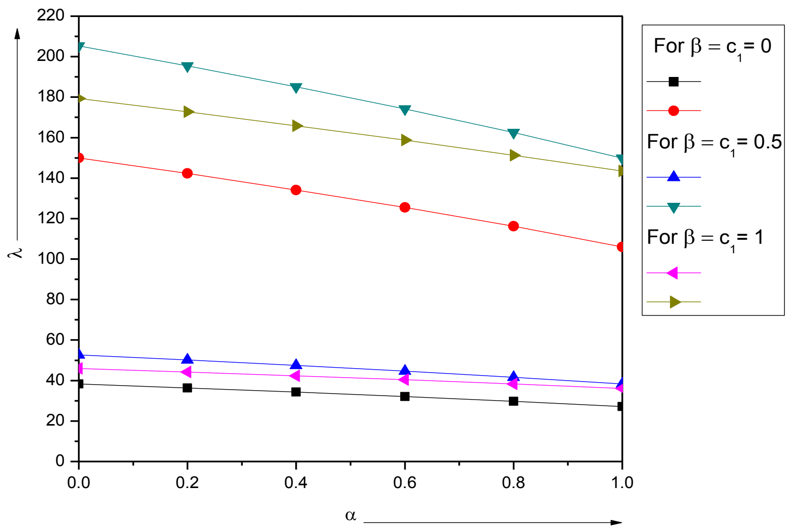

Figure 1: Thermal gradient vs frequency with three different values of taper constant (β = 0, 0.5, 1)

Table 2: Thermal gradient (α) versus frequency (λ) with fixed value of Poisson ratio (h = 0.345), three different values of taper constant (β = 0.2, 0.5, 1) and non-homogeneity constant (c_1= 0, 0.3, 0.6)

α	$C_1 = 0, \beta = 0.2$		$C_1 = 0.3, \beta = 0.5$		$C_1 = 0.6, \beta = 1$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	49.63	193.84	58.29	227.16	64.43	250.35
0.2	47.09	183.89	55.41	215.96	61.54	239.24
0.4	44.39	173.38	52.37	204.14	58.52	227.60
0.6	41.52	162.18	49.15	191.60	55.32	215.32
0.8	38.44	150.14	45.69	178.17	51.90	202.30
1.0	35.10	137.07	41.95	163.65	48.25	188.39

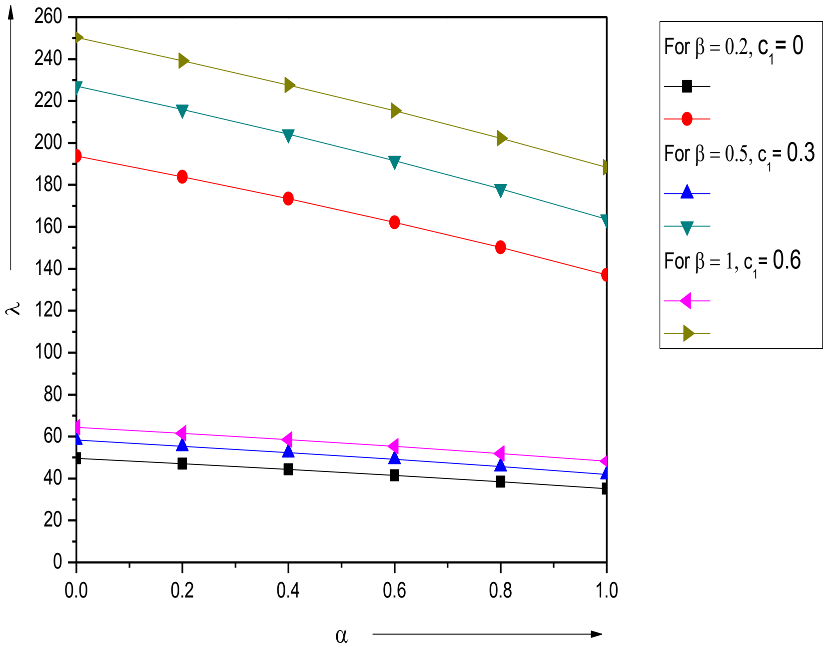

Figure 2: Thermal gradient vs frequency with three different values of taper constant (β = 0.2, 0.5, and 1) and non-homogeneity constant (c_1= 0, 0.3, and 0.6)

Table 3: Taper constant (β) versus Frequency (λ) with fixed value of Poisson ratio (*h* = 0.345) and three different values of thermal gradient (α = 0, 0.5, 1)

β	$C_1 = \alpha = 0$		$C_1 = \alpha = 0.5$		$C_1 = \alpha = 1$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	38.33	149.98	31.86	124.86	25.50	100.25
0.2	49.63	193.84	37.40	146.25	27.49	107.87
0.4	61.64	240.30	43.14	168.40	29.56	116.06
0.6	74.25	288.97	49.05	191.22	31.71	124.76
0.8	87.39	339.56	55.14	214.64	33.90	133.90
1.0	100.99	391.83	61.37	238.58	36.13	143.44

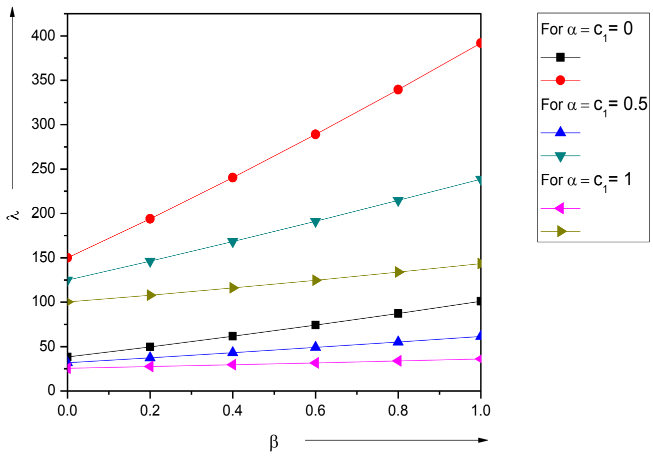

Figure 3: Taper constant vs frequency with three different values of thermal gradient (α = 0, 0.5, and 1)

Table 4: Taper constant (β) versus Frequency (λ) with fixed value of Poisson ratio (*h* = 0.345) and three different values of thermal gradient (α = 0.2, 0.5, 1) and non-homogeneity constant (c_1= 0, 0.3, 0.6)

β	C_1 = 0, α = 0.2		C_1 = 0.3, α = 0.5		C_1 = 0.6, α = 1	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	36.36	142.28	32.30	126.52	25.91	101.66
0.2	47.08	183.89	39.45	154.21	30.11	117.92
0.4	58.48	227.97	46.93	183.12	34.47	134.81
0.6	70.44	274.15	54.70	213.12	38.95	152.23
0.8	82.90	322.14	62.72	244.10	43.55	170.11
1.0	95.81	371.72	70.97	275.58	48.25	188.39

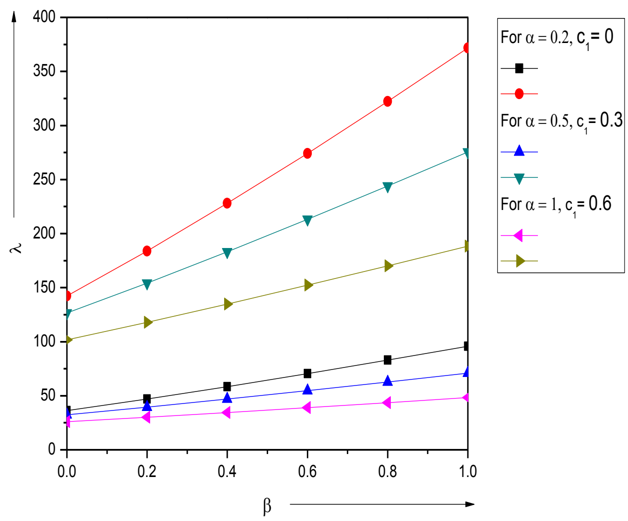

Figure 4: Taper constant vs frequency with three different values of thermal gradient (α = 0.2, 0.5, 1) and non-homogeneity constant (c_1= 0, 0.3, 0.6)

CONCLUSIONS

The present paper describes the behavior of frequencies for 1st and 2nd mode of vibration corresponding to the thermal gradient and taper constant. The frequency can be optimizing by taking suitable variation in parameters. The main objective of our study is to build up an arithmetical model for researchers and engineers so that they can use it for the advancement of technology with a realistic approach. Therefore engineers are advised to analyze our findings and develop the plate's structure in such a manner that can accomplish the basic requirements.

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