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Modelling and Analysis of Vibrations in Spherically Symmetric Transradially Isotropic Thermoelastic Sphere

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ABSTRACT: In this paper the study of vibration characteristics of spherically symmetric hollow sphere composed of an homogeneous thermoelastic material, based on the three dimensional coupled thermoelasticity has been studied. The frequency equations have been derived with the help of matrix Fröbenius series solutions obtained for the field functions. The frequency equations have been numerically simulated by using functional iteration method for hollow thermoelastic sphere made of magnesium material. The computer simulated results in respect of frequency and damping of vibration modes have been presented graphically.

Keywords: Spherically symmetric; transradial isotropic; thermoelastic; hollow sphere

INTRODUCTION

Practical applications of functional homogeneous materials are expected such as heat proof structural material. Hollow spherical tanks are used for storing hot and cold gases or liquids in petrochemical industries or propellent in rocket applications. Hollow spherical tanks find application because they have minimum application because they have minimum surface area for a given volume. Love [1] discussed the problems of vibration of isotropic sphere in detail. Dhaliwal and Singh [2] have explained similar problems in detail. Abd-Alla and Mahmoud [3] presented an analytical solution for magneto thermo-viscoelastic non-homogeneos medium with a spherical cavity subjected to periodic loading. Kanoria and Ghosh [4] studied the thermoelastic interactions in functionally graded spherically isotropic hollow spheres in which the thermo physical properties are temperature dependent in the context of linear theory of generalized thermoelasticity. Sharma and Sharma [5] investigated the distribution and stress in an infinite homogeneous transversely isotropic elastic solid having a spherical cavity by taking (i) unit step in stress and zero temperature change, and (ii) unit step in temperature and zero stress, at the boundary of the cavity. Bargi and Eslami [6] investigated the thermoelastic response of functionally graded hollow spheres based on Green-Lindsay theory of thermoelasticity. Chen [7] investigated some problems in spherically isotropic elastic materials. Sharma and Sharma [8,9] investigated the free vibrations of homogeneous transradially isotropic coupled and generalized thermoelastic spheres by using matrix Frobenius method.Sharma et al. [10] studied the analysis of axisymmetric functionally graded viscothermoelastic spheres.

Formulation: Consider a homogeneous, transradially isotropic, thermally conducting, spherically symmetric elastic sphere of radius R at uniform temperature T_0 in the undisturbed state initially. For the plain strain problem, the components of displacement in spherical coordinate system are expressed as $\vec{u} = (u_r, 0, 0)$ and $\partial/\partial\theta \cong 0 = \partial/\partial\phi$ respectively. The governing equations of motion and heat conduction in the absence of body forces and heat sources, and also taking the symmetry about radial direction for linear theory of coupled thermoelasticity are given by

$$\sigma_{rr,r} + \frac{1}{r}\left[2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi}\right] = \rho\ddot{u}_r \qquad (1)$$

$$K_3\left(T_{,rr} + \frac{2}{r}T_{,r}\right) - \rho C_e\dot{T} = T_0\left[\beta_1\left(\dot{e}_{\theta\theta} + \dot{e}_{\phi\phi}\right) + \beta_3 e_{rr}\right] \quad (2)$$

$$\sigma_{\theta\theta} = c_{11}e_{\theta\theta} + c_{12}e_{\phi\phi} + c_{13}e_{rr} - \beta_1 T,$$

$$\sigma_{\phi\phi} = c_{12}e_{\theta\theta} + c_{11}e_{\phi\phi} + c_{13}e_{rr} - \beta_1 T$$

$$\sigma_{rr} = c_{13}\left(e_{\theta\theta} + e_{\phi\phi}\right) + c_{33}e_{rr} - \beta_3 T, e_{rr} = \frac{\partial u_r}{\partial r},$$

$$e_{\theta\theta} = \frac{u_r}{r}, \; e_{\phi\phi} = \frac{u_r}{r} \qquad (3)$$

where

$$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3, \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3, \qquad (4)$$

Upon using equations (1) - (2) via equations (3)-(4), we obtain

$$\frac{\partial^2 U_\zeta}{\partial\zeta^2} + \frac{2}{r}\frac{\partial U_\zeta}{\partial\zeta} + \frac{2(c_3 - c_2 - c_1)}{\zeta^2}U_\zeta - \beta^*\left[\frac{\partial\Theta}{\partial\zeta} + \frac{2}{\zeta}\left(1 - \bar{\beta}\right)\Theta\right] = \frac{\partial^2 U_\zeta}{\partial\tau^2} \quad (5)$$

$$\left(\frac{\partial^2}{\partial\zeta^2} + \frac{2}{r}\frac{\partial}{\partial\zeta}\right)\Theta - \Omega^*\frac{\partial\Theta}{\partial\tau} - \varepsilon^*\Omega^*\frac{\partial}{\partial\tau}\left(\frac{\partial}{\partial\zeta} + \frac{2\bar{\beta}}{\zeta}\right)U_\zeta = 0 \quad (6)$$

Here we have defined and used the dimensionless quantities

$\zeta = r/R, \quad \tau = v_s t/R, \; U_\zeta = u_r/R, \; T' = T/T_0 \;, \; \tau_{ij} = \sigma_{ij}/c_{33},$

$\zeta_1 = a/R,\ \zeta_2 = b/R\ c_1 = c_{11}/c_{33},\ \ c_2 = c_{12}/c_{33},$

$c_3 = c_{13}/c_{33}\ \ \ \varepsilon = T_0\beta_3{}^2/\rho C_e c_{33}\ ,\ R = \omega^{\bullet}R/v_p$

$$\varepsilon^* = \varepsilon/\beta^*,\ \beta^{\bullet} = \beta_3 T_0/c_{33},\ \Omega^* = \omega^* R/v_p \tag{7}$$

where $v_p{}^2 = c_{33}/\rho$ and $\omega^{\bullet} = C_e c_{33}/K_3$ are longitudinal wave velocity and characteristic frequency of the sphere respectively. Here a and b are the inner and outer radius of the hollow sphere. The primes have been suppressed for convenience.

hollow sphere is subjected to two types of boundary conditions at its inner surface $(\zeta = \zeta_1)$ and outer surface $(\zeta = \zeta_2)$.

Given as,

$U_\zeta = 0,$

$$T_{,\zeta} = 0 \text{ at } \zeta = \zeta_1,\ \zeta = \zeta_2 \tag{8}$$

Solution of the problem: We assume the solution as

$$(U_\zeta(\zeta,\tau),\Theta(\zeta,\tau)) = \zeta^{-\frac{1}{2}}(W(\zeta),\Theta^*(\zeta))\exp(i\Omega\tau) \tag{9}$$

The substitution of equations (9) into equations (5) and (6) provides us

$$\left(\nabla_2^2 + 1 - \frac{a_3^{*2}}{\xi^2}\right)W - \Omega^{-1}\beta^*\left(\frac{\partial}{\partial\xi} + \frac{3-4\overline{\beta}}{2\xi}\right)\Theta^* = 0 \tag{10}$$

$$\left(\nabla_2^2 - i\Omega^{-1}\Omega^* - \frac{1}{4\xi^2}\right)\Theta^* + i\Omega^*\varepsilon^*\left(\frac{\partial}{\partial\xi} + \frac{4\overline{\beta}-1}{2\xi}\right)W = 0 \tag{11}$$

where

$a_3^{*2} = (1 + 8(c_1 - c_3 + c_2))/4,\ \ \nabla_2^2 = \frac{d^2}{d\xi^2} + \frac{1}{\xi}\frac{d}{d\xi}$,

$\xi = \zeta\,\Omega$

In equations (10)-(11), ξ = 0 (or $\zeta = 0$) is a regular singular point of each constituent equation. Moreover all the coefficients are finite, single valued and continuous in the considered regions. Thus, the equations (10)-(11) satisfy the necessary conditions of Fröbenius method and hence we can make use of the Fröbenius series method to solve this equation.

We assume series solution

$$\mathbf{Z}^* = \sum_{k=0}^{\infty} \tilde{Z}_k \xi^{s+k} \text{ Where } \tilde{\mathbf{Z}}_\mathbf{k} = [A_k \quad C_k]' \tag{12}$$

Upon using solution (12) in equation (10)-(11) and simplifying, we get

$$\sum_{k=0}^{\infty}\left[\mathbf{H_1^*}(\mathbf{s}+\mathbf{k})\xi^{-2} + \mathbf{H_2^*}(\mathbf{s}+\mathbf{k})\xi^{-1} + \mathbf{H}^*\right]\xi^{s+k}\tilde{Z}_k = 0 \tag{13}$$

where

$\mathbf{H_1^*}(\mathbf{s}+\mathbf{k}) = diag\ ((s+k)^2 - a_3^{*2}, (s+k)^2 - \frac{1}{4}),$

$\mathbf{H}^* = diag(1, -i\Omega^{-1}\Omega^*)$

$$\mathbf{H_2^*}(\mathbf{s}+\mathbf{k}) = \begin{bmatrix} 0 & \Omega^{-1}\beta^*\left(s+k+\frac{3-4\overline{\beta}}{2}\right) \\ i\Omega^*\varepsilon^*\left(s+k+\frac{4\overline{\beta}-1}{2}\right) & 0 \end{bmatrix} \tag{14}$$

Equating the coefficients of lowest powers of ξ (i.e. the coefficient of ξ^{s-2}) to zero in equation (13), we obtain:

$$\mathbf{H_1^*}(\mathbf{s})\tilde{Z}_0 = 0 \tag{15}$$

The system of equations (15) will have a non-trivial solution if and only if

$|H_1^*(s)| = 0$. This leads to the indicial equation whose solution (eigenvalues) is given as

$$s_1 = a_3^*, s_2 = \frac{1}{2}, s_3 = -a_3^*, s_4 = -1/2 \tag{16}$$

The corresponding eigenvectors are given by

$\tilde{\mathbf{Z}}_\mathbf{0} = \begin{bmatrix} A_0 \\ C_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\tilde{N}_0$ where $\tilde{N}_0$ is constant.

Again the equation (13) implies that

$$\tilde{\mathbf{Z}}_{\mathbf{k+2}} = -\mathbf{H_1^{*-1}}(\mathbf{s_j}+\mathbf{k}+\mathbf{2})\left[\mathbf{H_2^*}(\mathbf{s_j}+\mathbf{k}+\mathbf{1})\tilde{\mathbf{Z}}_{\mathbf{k+1}} + \mathbf{H}^*\tilde{\mathbf{Z}}_\mathbf{k}\right] \tag{17}$$

Now putting $k = 0, 1, 2, 3......$ in equation (17) successively and simplifying.

Continuing in this manner and simplifying, we get

$$\tilde{\mathbf{Z}}_{\mathbf{2k+2}}(\mathbf{s_j}) = \mathbf{W_{2k+2}^*}(\mathbf{s_j})\tilde{\mathbf{Z}}_\mathbf{0} \tag{18}$$

$$\tilde{\mathbf{Z}}_{\mathbf{2k+1}}(\mathbf{s_j}) = -\mathbf{W_{2k+1}^*}(\mathbf{s_j})\tilde{\mathbf{Z}}_\mathbf{0} \tag{19}$$

On simplification we have

$$\mathbf{W_{2k+2}^*} \approx o\left(\frac{1}{k}\right)\mathbf{W}^*,\quad \mathbf{W_{2k+1}^*} \approx o\left(\frac{1}{k}\right)\mathbf{W}^{**} \tag{20}$$

where

$\mathbf{W}^* = \frac{1}{2}\begin{bmatrix} \beta^*\Omega^{-1}+1 & 0 \\ 0 & i\varepsilon^*\Omega^*\beta^* \end{bmatrix}$ and

$$\mathbf{W}^{**} = \frac{1}{2}\begin{bmatrix} 0 & -\beta^*\Omega^{-1} \\ i\varepsilon^*\Omega^* & 0 \end{bmatrix} \tag{21}$$

Thus both the matrices $\mathbf{W_{2k+2}^*} \to 0$ and $\mathbf{W_{2k+1}^*} \to 0$, as $k \to \infty$.

Now according to Cullen [11], a sequence $\{W_k\}$ of matrices converges to a matrix $\mathbf{W}$ as $k \to \infty$, that is $\underset{k\to\infty}{Lim} W_k = W$, if each k^2 component sequence converges. Thus the formal solution (12) in case hollow spheres becomes:

$$W = \sum_{k=0}^{\infty}[E_1 w_{11}^{2k+2}(s_1)(\xi)^{s_1+1} - E_2 w_{12}^{2k+1}(s_2)(\xi)^{s_2} + E_3 w_{11}^{2k+2}(-s_1)(\xi)^{-s_1+1} - E_4 w_{12}^{2k+1}(-s_2)(\xi)^{-s_2}](\xi)^{2k+1}$$

$$\Theta^* = \sum_{k=0}^{\infty}[-E_1 w_{21}^{2k+1}(s_1)(\xi)^{s_1} + E_2 w_{22}^{2k+2}(s_2)(\xi)^{s_2+1} - E_3 w_{21}^{2k+1}(-s_1)(\xi)^{-s_1} + E_4 w_{22}^{2k+2}(-s_2)(\xi)^{-s_2+1}](\xi)^{2k+1} \quad (22)$$

SECULAR EQUATIONS

We assume that for a radially isotropic, thermally conducting sphere the rigidly fixed and thermally insulated boundary condition hold.

$\mathbf{D}^*\mathbf{X}^* = 0$,

where $\mathbf{X}^* = [E_1 \quad E_2 \quad E_3 \quad E_4]^T$,

$$\mathbf{D}^* = \left(d_{ij}^{*k}\right)_{4\times4} \quad (23)$$

$d_{11}^{*k} = w_{11}^{2k+2}(s_1)(\Omega\zeta_1)^{s_1+1}$, $d_{12}^{*k} = -w_{12}^{2k+1}(s_2)(\Omega\zeta_1)^{s_2}$

, $d_{13}^{*k} = w_{11}^{2k+2}(-s_1)(\Omega\zeta_1)^{-s_1+1}$

$d_{14}^{*k} = -w_{12}^{2k+1}(-s_2)(\Omega\zeta_1)^{-s_2-1}$

$d_{31}^{*k} = -(s_1 + 2k + 1)w_{21}^{2k+1}(s_1)(\Omega\zeta_1)^{s_1-1}$,

$d_{32}^{*k} = (s_2 + 2k + 2)w_{22}^{2k+2}(s_2)(\Omega\zeta_1)^{s_2}$

$d_{33}^{*k} = -(-s_1 + 2k + 1)w_{21}^{2k+1}(-s_1)(\Omega\zeta_1)^{-s_1-1}$,

$$d_{34}^{*k} = (-s_2 + 2k + 2)w_{22}^{2k+2}(-s_2)(\Omega\zeta_1)^{-s_2} \quad (24)$$

Here the elements d_{ij}^{*k} $(i = 2, 4;\ j = 1, 2, 3, 4)$ can be written from d_{ij}^{*k} $(i = 1, 3;\ j = 1, 2, 3, 4)$ by replacing ζ_1 with ζ_2 therein.

The equation (23) has a non-trivial solution if and only if

$$\det(\mathbf{D}^*) = |\mathbf{D}^*| = 0 \quad (25)$$

This leads to a 4×4 determinantal frequency equation that governs the free vibrations in this case.

RESULTS AND DISCUSSION

In order to illustrate the analytical developments in the previous sections, we now perform some numerical computations and simulations. The secular equation (25) contains complete information about the effect of different fields lowest frequency, thickness to mean radius ratio ($t^* = \dfrac{\zeta_2 - \zeta_1}{(\zeta_2 + \zeta_1)/2}$) and damping factor of hollow sphere. The results presented graphically for Magnesium material.

The material parameters and constants for these crystals used in numerical computations are given in tables 1.

Table 1: Physical data for magnesium crystal (Dhaliwal and Singh [2])

$c_{11} = 5.974\times10^{10}\ Nm^{-2}$, $c_{12} = 2.624\times10^{10}\ Nm^{-2}$, $c_{13} = 0.508\times10^{11}\ Nm^{-2}$, $c_{33} = 0.627\times10^{11}\ Nm^{-2}$, $c_{44} = 3.278\times10^{10}\ Nm^{-2}$, $\beta_1 = 5.75\times10^6\ Nm^{-2}\deg^{-1}$, $\beta_3 = 5.17\times10^6\ Nm^{-2}\deg^{-1}$, $K_1 = 1.7\times10^2\ Wm^{-1}\deg^{-1}$, $\omega^* = 3.58\times10^{11}$, $\rho = 7.14\times10^3\ kgm^{-3}$, $C_e = 3.9\times10^2\ J\,kg^{-1}\deg^{-1}$

The numerical computations have been performed by employing the procedure outlined in Sharma and Sharma [5] to the dispersion relation (25) with the help of MATLAB programming. The computations have been done for different values of thickness to mean radius ratio.

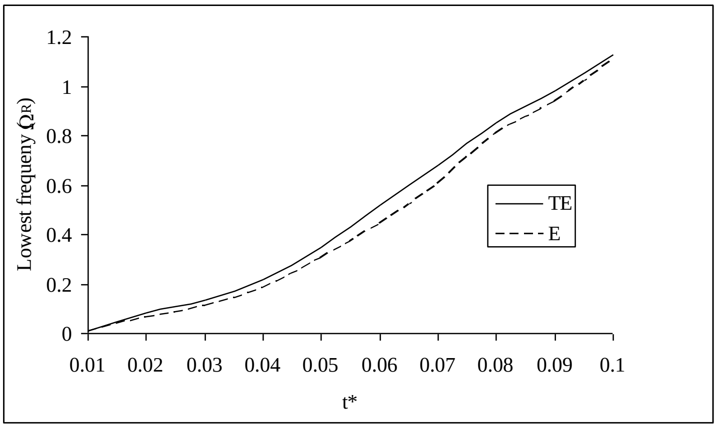

Figure 1 Lowest frequency versus thickness to mean radius ratio (t*) in magnesium material hollow sphere

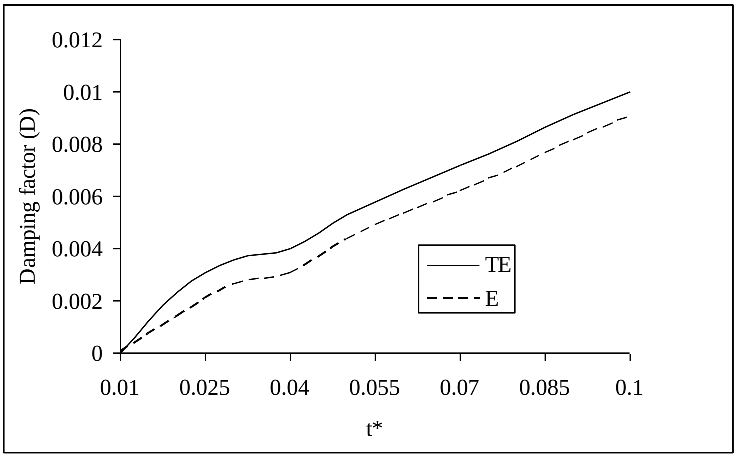

Figure 2 Damping factor versus thickness to mean radius ratio (t*) in magnesium material hollow sphere

Figure 1 represents the variations of lowest frequency (Ω_R) with thickness to mean radius ratio of rigidly fixed thermally insulated t^* hollow sphere of magnesium material . The magnitude of lowest frequency increases with the increase of t*. In Figure 1 it is noticed that for $0.01 < t^* < 0.03$ the lowest frequency is found to be almost constant and after this it increases linearly with the increase of t^*. Also it is found that the vibrations in case of thermoelastic field more than that of in case of elastic field for both the materials.

Figure 2 represents the variations of damping factor (D) with thickness to mean radius ratio t* for magnesium material. For magnesium material the magnitude of damping factor increases monotonically in the region $0.01 < t^* < 0.045$ and for t^*>0.045, it increases linearly for elastic and thermoelastic case. Also the magnitude of damping is greater in presence of thermoelastic field in comparison to elastic field.

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